

CALCULATION OF HUMIDITY PARAMETERS AND UNCERTAINTIES USING DIFFERENT FORMULATIONS AND SOFTWARES

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ABSTRACT

The increase of importance of humidity instruments in the last 10 years brought their applications to several parts of industry, like automotive, pharmaceutical, food and laboratories to comply with regulations and quality requirements. As consequence, there has been a growing interest of manufacturers and secondary laboratories in having more accurate humidity measurements.. That was the moment to consider performance evaluation and calibration issues. A humidity calibration laboratory should pay a special attention to temperature effects on humidity measurements, in order to reduce their uncertainties. Nowadays, there are several commercially available softwares that allow calculation of psychrometer properties, such as vapor pressure (dew point), absolute humidity and relative humidity. These softwares are used by secondary laboratories during their calibrations and can be one of the sources of uncertainty measurements. This paper calculates psychrometer properties for humidity measurements by using the state of the art formulation, and softwares. Then calculates uncertainties for different conditions.

1. INTRODUCTION

Recently, in Brazil, there has been a good interest for humidity measurement, mainly for quality control of industrial processes, like pharmaceutical, food and automotive, where air conditioning plays an important role. The National Metrology Institute (INMETRO) is in charge of keeping the humidity standards updated and calibrates most of the humidity instruments in Brazil. There is only one accredited laboratory in the Brazilian Calibration Network (RBC) for calibration services, but there has been a good interest among other laboratories to be part of the network. The national humidity standard is a dew point apparatus, inside a temperature and humidity controlled industrial climatic chamber. Thus, there is no standard humidity generator for calibrating humidity standards, which must be calibrated abroad, when higher accuracy is required. Many laboratories measure humidity by less accurate methods, relying both on expressions for calculating humidity from basic measured parameters and commercially available softwares. Thus, there is a need for comparing the results obtained from the available formulations, including those normally used for gas mixture property calculation, and thus estimating the uncertainty of measurement.

2. THEORETICAL BACKGROUND

2.1 State of the art of humidity calibration

The uncertainty of relative humidity measurement in the standard humidity generator of three national institutes [18] has been estimated, together with its measurement made by a dew point hygrometer. Relative humidity is defined as the ratio between the partial pressure of water vapor in a given sample of moist air, and the partial pressure of water vapor in the same sample of mist air if it were saturated mixture at the same temperature and pressure.

- (1) NMi Van Swinden Laboratorium (NMi-VSL), Delft, The Netherlands – Uses a two-pressure and single temperature humidity generator, measuring the dew point with a

calibrated instrument. In this experimental procedure, a saturated gas under pressure is , firstly, brought isothermally to a lower pressure by means of an expansion valve, and then fed directly to the sensor under calibration. The ratio between the pressures determines the output partial vapour pressure, from which the dew point can be calculated and compared to the measured one. This humidity generator is used only as a stable humidity source.

- (2) Measurements Standards Laboratory, Industrial Research Ltd, New Zealand – Uses a two-pressure and single temperature humidity generator. In this experimental procedure, a saturated gas under pressure is , firstly, brought isothermally to a lower pressure by means of an expansion valve, and then fed directly to the sensor under calibration. The ratio between the pressures determines the output partial vapour pressure, from which the dew point can be calculated.
- (3) National Physical Laboratory, UK – Uses a single pressure and a single temperature humidity generator. In this experimental procedure, a gas is saturated at a given temperature, and then, fed directly to the sensor under calibration. The saturator temperature is directly related to the dew point.

The uncertainty budget is then detailed for the following measured parameters of a relative humidity hygrometer calibration. Table 1 shows the results.

- Saturation temperature
- Saturation pressure
- Pressure near sensor
- Temperature near sensor
- Measured dewpoint
- Flow measurement
- Vapour pressure and enhancement factor formulations

Table 1 . Relative humidity hygrometer calibration by national laboratories

Laboratory	Temperature °C	Relative Humidity % rh	Expanded Uncertainty (k=2) ± % rh
MSL	20,000	50,00	0,25
NMi-VSL	20,000	50,00	0,38
NPL	20,000	80,00	0,42

Other significant sources of uncertainty in humidity calibrations include reproducibility, hysteresis and response time. Also, the fitting influences the uncertainty. MSL [18] calibrated a RH-sensor in the 17 °C to 23 °C temperature range, and 15 % rh to 70 % rh relative humidity range, and obtained a ± 0,52 % rh uncertainty for a cubic correction equation and ± 0,70 % rh for a linear correction equation.

2.2 Governing equations

The pure-phase saturation vapour pressure of water or ice at a temperature t is usually denoted $e(t)$. The partial pressure P_v of water vapour in saturated mist air at t is greater than $e(t)$. The so-called water vapour enhancement factor $f(t,P)$ is a function of both temperature and pressure.

$$f(t, P) = \frac{P_v}{e(t)} \quad (1)$$

2.2.1 Relative humidity

The relative humidity (h) near the sensor under test is calculated from the vapour mole fraction x_o and from the air temperature t_o and pressure P_o near the sensor, using the equation,

$$h = \frac{x_o \cdot P_o}{f(t_o, P_o) \cdot e(t_o)} \cdot 100 \text{ (\% rh)} \quad (2)$$

For two-temperature and two-pressure humidity generation, the vapour mole fraction is calculated from measurements of saturation pressure P_s and temperature t_s , using

$$x_o = \frac{f(t_s, P_s) \cdot e(t_s)}{P_s} \quad (3)$$

For dew point measurement calibration, the same equation may be used but now t_s represents the measured dew point temperature, and P_s represents the pressure at the dew point temperature sensor.

Various expressions for $e(t)$ and $f(t,P)$ have been published These have been reviewed in Boyes & Bell [3], Hardy [6], NPL-IMC [15], Sonntag [16,17].

2.2.2 Water vapour pressure formulation

The most commonly used vapour pressure formulations are given by Wexler [23,24]. They were updated to ITS-90 by Sonntag [17] and Hardy [6]. Huang [8] also published an updated version of Wexler's equation. Wagner & Pruss [19,20] presented new formulations for vapour pressure above water and ice, respectively. Finally, the less accurate equation is due to Magnus [15].

Sonntag's formulation [16] is the mostly used one, according to Equation 4. Table 2 presents the coefficients and un

$$e_s = \exp\left(\sum_{i=1}^4 a_i T^{i-2} + a_7 \ln(T)\right) \quad (4)$$

Table 2 : Sonntag's formulation [16] for water vapour pressure above water and ice

Coefficient	Above water	Above ice
a_1	-6,0969385E+03	-6,0245282E+03
a_2	2,12409642E+01	2,932707E+01
a_3	-2,711193E-02	1,0613868E-02
a_4	1,673952E-05	-1,3198825E-05
a_7	2,433502	-4,9382577E-01
Standard Uncertainty	< ± 0,005 % rh	< ± 0,5 % rh

2.2.3 Water vapour pressure enhancement factor formulations

The most accurate formulation for $f(t,P)$ is generally accepted to be that given by Greenspan [4], according to [18], based on a correlation published by Hyland [10]. The formulation covers the -100 °C to +100 °C temperature range, and 1 to 20 bar pressure range. Table 3 presents the

coefficients.

$$f(t, P) = \exp \left[\alpha(t) \cdot \left(1 - \frac{e(t)}{P} \right) + \beta(t) \cdot \left(\frac{P}{e(t)} - 1 \right) \right] \quad (5)$$

$$\alpha(t) = \sum_{i=1}^4 A_i \cdot (t + 273,15)^i \quad (6)$$

$$\beta(t) = \exp \left[\sum_{i=1}^4 B_i \cdot (t + 273,15)^i \right] \quad (7)$$

Table 3 : Coefficients of the water vapour enhancement factor formulation [15]

Coefficient	Bellow triple point	Above triple point
A ₁	3,64449E-04	3,53624E-04
A ₂	2,93631E-05	2,93228E-05
A ₃	4,88635E-07	2,61474E-07
A ₄	4,36543E-09	8,57538E-09
B ₁	-1,07271E+01	-1,07588E+01
B ₂	7,61989E-02	6,32529E-02
B ₃	-1,74771E-04	-2,53591E-04
B ₄	2,46721E-06	6,33784E-07

The standard uncertainty for the enhancement factor (u_f) follows three equations according to the temperature and pressure ranges. The coefficients are given in Table 4

$$u_f = 1,05 \cdot (a_0 \cdot P + a_1) \cdot \exp[(a_2 \cdot \ln(P) + a_3) \cdot t] \quad \text{for } t < 0 \text{ } ^\circ\text{C} \quad (8)$$

$$u_f = 1,05 \cdot (b_0 \cdot P + b_1) \cdot \exp[(b_2 \cdot \ln(P) + b_3) \cdot t] \quad \text{for } t > 31 \text{ } ^\circ\text{C} \text{ and } P > 4 \text{ bar} \quad (9)$$

$$u_f = 1,05 \cdot (c_0 \cdot P + c_1) \cdot t + c_2 \cdot P + c_3 \quad \text{for } t < 31 \text{ } ^\circ\text{C} \text{ and } P = 4 \text{ bar} \quad (10)$$

Table 4 : Coefficients for the determination of standard uncertainty in the enhancement factor

Equation (8)		Equation (9)		Equation (10)	
a ₀	1,04E-04	b ₀	6,4E-05	c ₀	-2,4557E-07
a ₁	-1,0E-04	b ₁	1,0E-04	c ₁	-5,6041E-07
a ₂	0	b ₂	6,45E-03	c ₂	6,355E-05
a ₃	-1,51E-02	b ₃	-2,59E-02	c ₃	9,452E-06

2.2.4 Dew point from pure vapour pressure

According to [15], the following equation relates dew point (t_d) and pure vapour pressure (e)

$$t_d = \frac{243,12 \cdot \ln(e/611,2)}{17,62 - \ln(e/611,2)} \quad \text{above water} \quad (11)$$

with an expanded uncertainty of less than $\pm 0,04$ °C, in the -45 °C to 60 °C temperature range, and

$$t_d = \frac{272,62 \cdot \ln(e/611,2)}{22,46 - \ln(e/611,2)} \quad \text{above ice} \quad (12)$$

with an expanded uncertainty of less than $\pm 0,08$ °C, in the -65 °C to 0,01 °C temperature range

2.2.5 Moisture content (g) from dew point measurement

After having measured dew point, Equations (11) and (12) can be used to determine pure vapour pressure (e). The moisture content (g) can be calculated [15] as :

$$g = \frac{0,62197 \cdot f \cdot e}{P_{at} - f \cdot e} \quad (13)$$

2.2.6 Uncertainty calculations

The uncertainty of relative humidity and moisture content as a function of dew point was calculated according to the methods of ISO GUM [11]. The sensibility coefficients were numerically evaluated.

2.2.7 Softwares

Four (4) softwares, available through Internet, were tested. All of them use Sonntag's [16] formulation. Therefore, a comparison between their output and Sonntag's values were made.

- Society of Environmental Engineers [26]
- General Eastern Instruments Co.[27]
- Michell Instruments [28]
- Thunder Scientific [29]

3. RESULTS

3.1 Air composition

The numerical value in Equation (13) is the ratio between molecular weights of water and dry air, thus depending on air composition. The following references were consulted. Table 5 presents different dry air compositions and the molecular weight ratio for the following references :

- (1) Jones [12]
- (2) ANSI/ASHRAE [1]
- (3) Chemie [30]
- (4) Columbia [31]
- (5) Engineering Tools [32]
- (6) ChemWeek [33]
- (7) Air Teachers [34]

Table 5 : Dry air molar composition (%) and molecular weight ratio (MWR)

Constituent	(1)	(2)	(3)	(4)	(5)	(6)	(7)
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N ₂	78,030	77,984	78,090	78,084	78,090	78,080	78,090
O ₂	20,990	20,9476	20,930	20,947	20,950	20,950	20,940
Ar	0,94	0,934	0,940	0,934	0,933	0,930	0,930
CO ₂	0,030	0,0314	0,030	0,0350	0,030	0,033	0,032
Ne		0,001818		0,001818	0,00180	0,0018	0,0018
He		0,000524		0,000524	0,0005	0,00052	0,0052
CH ₄		0,00015		0,00017	0,0001	0,0002	0,00015
Kr		0,000114		0,000114		0,00011	0,0001
H ₂	0,010	0,00005	0,010	0,000053			0,00005
N ₂ O		0,00005		0,000031		0,00005	0,0000001
Xe		0,0000087		0,0000087			0,000008
M (water)	18,02	18,01534	18,02	18,02	18,02	18,02	18,02
M(dry air)	28,969	28,964	28,967	28,971	28,971	28,968	28,967
MWR	0,62206	0,62198	0,62209	0,62200	0,62201	0,62206	0,62208

It can be seen that the ratio for different compositions is $(0,62204 \pm 0,00006)$. The difference for NPL's value is 0,011 %, which is smaller than uncertainty value.

3.2 Software performance

Figures 1, 2, 3 and 4 shows the relative humidity difference between the software value and Sonntag's formulation in the 0 to 90 °C dew point and moist air ranges. It can be seen that all data stay in the $\pm 0,02$ % rh range, with a few exceptions. Michell's and Thunder Scientific's software provide better agreement, to within $\pm 0,01$ % rh. Anyway, this difference is much smaller than the uncertainty of measurement, and the softwares can be used for relative humidity calculations.

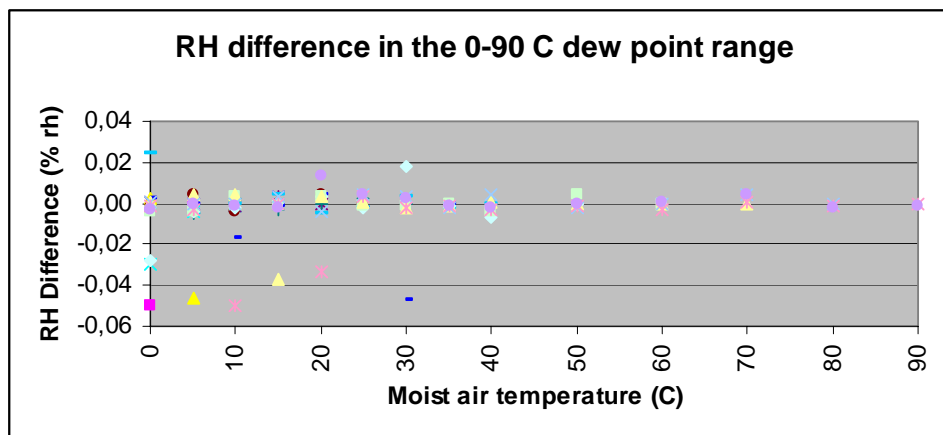


Figure 1 : Relative Humidity difference between Society of Environmental Engineers' software [26] and Sonntag's formulation [16],

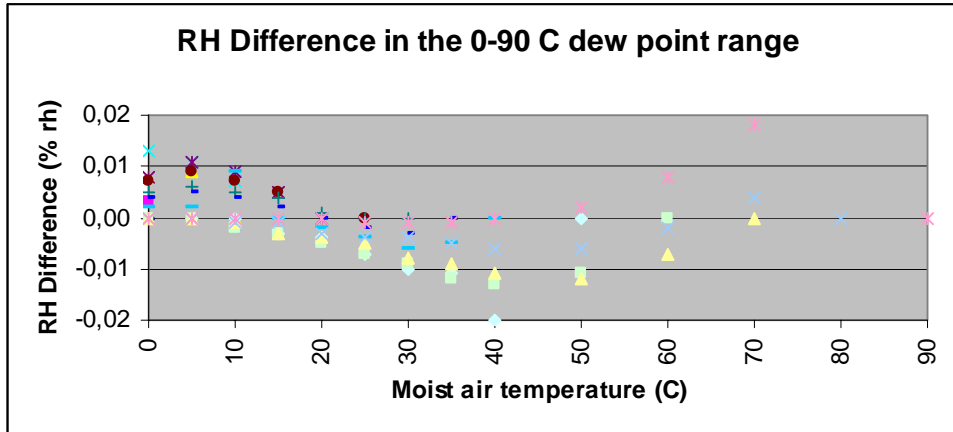


Figure 2 : Relative Humidity difference between General Eastern Instrument Co.'s software [27] and Sonntag's formulation [16]

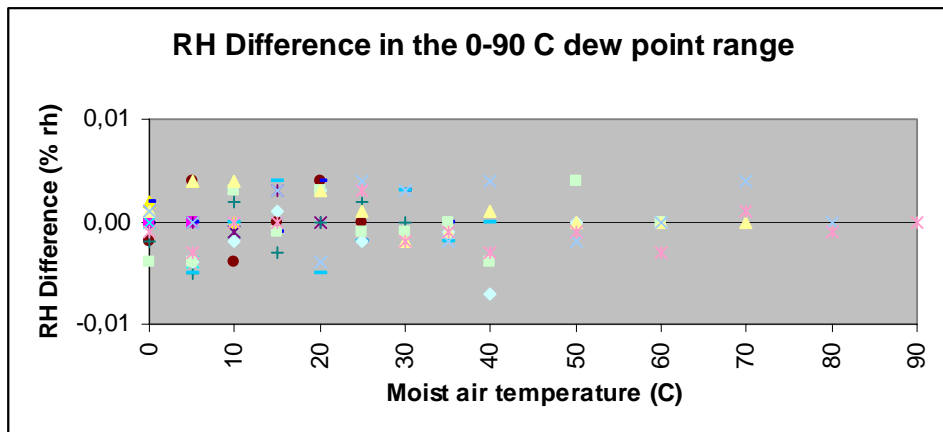


Figure 3 : Relative Humidity difference between Michell's software [28] and Sonntag's formulation [16]

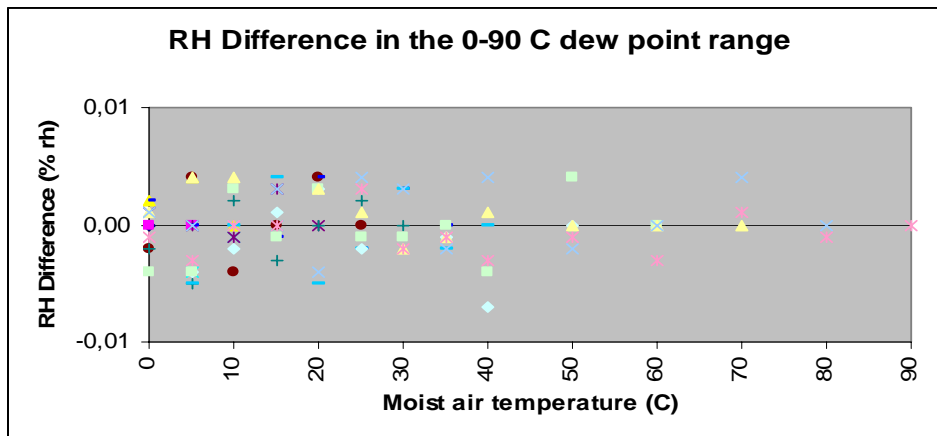


Figure 4 : Relative Humidity difference between Thunder Scientific's software [29] and Sonntag's formulation [16]

3.3 Relative Humidity and Moisture Content Expanded Uncertainty (k=2) as a function of dew point

The uncertainty was calculated for relative humidity and moisture content for the same temperature as in [18]. It can be seen that the relative humidity uncertainty is larger than the values obtained in the national laboratories because the dew point and moist air uncertainties are larger. So, by reducing these values, smaller uncertainty values can be obtained.

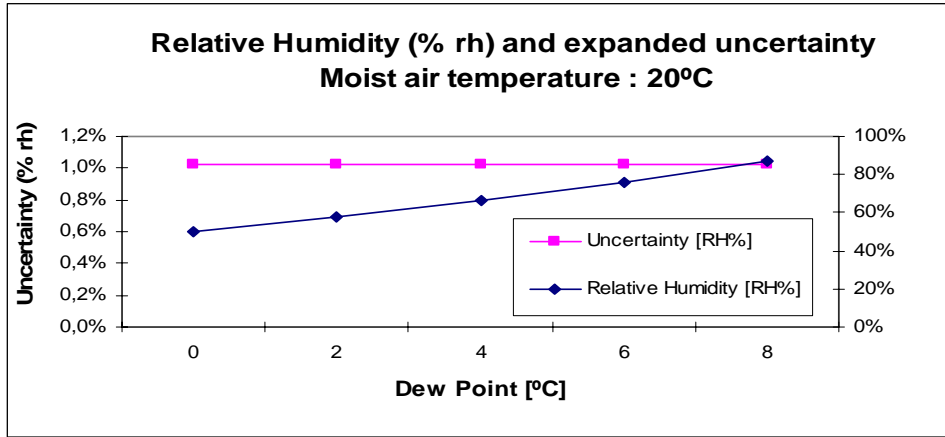


Figure 5 : Relative humidity uncertainty for $\pm 0,2$ °C dew point uncertainty and $\pm 0,1$ °C moist air temperature uncertainty

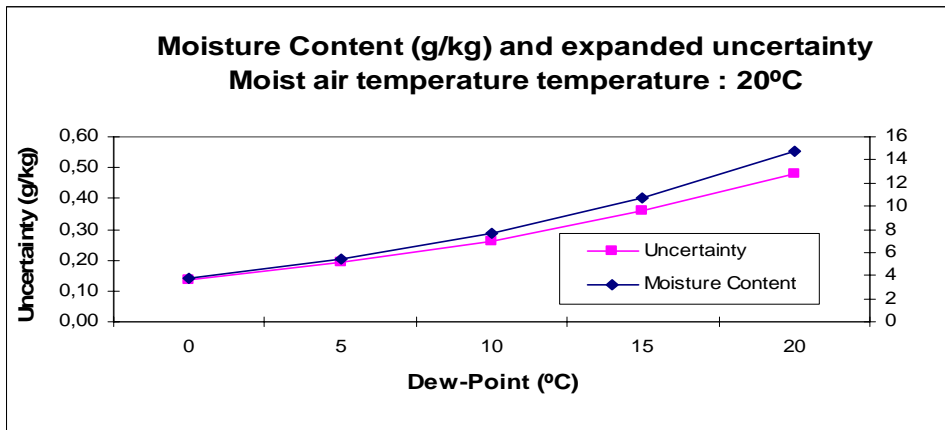


Figure 6 : Moisture Content uncertainty for $\pm 0,2$ °C dew point uncertainty and $\pm 0,1$ °C moist air temperature uncertainty

4. CONCLUSIONS

An uncertainty propagation study for the measurement of relative humidity and moisture content shows that if smaller uncertainty values are to be obtained, the dew point measurement uncertainty must be below $\pm 0,2$ °C, which is compatible with many dew point instruments in laboratories ($\pm 1,0$ % rh). This study, thus, shows the upper limit for measurement uncertainty in calibration laboratories.

Several available softwares were tested, resulting in differences with respect to Sonntag's formulation in less than $\pm 0,02$ % rh, which is much smaller than relative humidity uncertainty.

Thus, they can be used for humidity calculations.

The influence of the dry air composition, according to several authors, was shown to be much smaller than the measurement uncertainty of moisture content. Thus, there is no need of improving its determination

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